Bonus 4: Solution

Since we can't use product rule, we must go back to the definition of the derivative. So we have:

$$f'(0) = \lim_{h \to 0} \frac{f(0+h) - f(0)}{h}$$

But we know $f(x) = xg(x)$, so we have

$$f'(0) = \lim_{h \to 0} \frac{hg(h) - 0g(0)}{h}$$

Simplifying, we get

$$f'(0) = \lim_{h \to 0} \frac{hg(h)}{h} = \lim_{h \to 0} g(h)$$

But since g is continuous at 0, we know that

$$f'(0) = \lim_{h \to 0} g(h) = g(0)$$

So we know that $f'(0)$ exists and equals $g(0)$.